Inference at $*$ 1 0
of proof for Lemma minus_imax:

1. a : $\mathbb{Z}$
2. b : $\mathbb{Z}$

$\vdash$ (-if a $\leq$z b then b else a fi) = if (-a) $\leq$z -b then -a else -b fi
by PERMUTE{1:n,
2:n,
3:n,
4:n,
5:n,
6:n,
7:n,
8:n,
9:n,
10:n,
11:n,
12:n,
13:n,
14:n,
15:n,
16:n,
17:n,
18:n,
19:n,
20:n,
18:n,
21:n,
19:n,
22:n,
23:n,
24:n,
25:n,
26:n,
24:n,
23:n,
27:n,
28:n,
29:n,
30:n,
28:n,
31:n,
29:n,
32:n,

33:n,
34:n,
35:n,
36:n,
34:n,
33:n,
37:n,
38:n,
39:n,
40:n,
41:n,
42:n,
43:n,
44:n,
45:n,
46:n,
47:n,
48:n,
46:n,
49:n,
47:n,
50:n,
51:n,
52:n,
53:n,
54:n,
52:n,
51:n,
55:n}

1: .....wf..... NILNIL

$\vdash a \leq\text{z } b \in \mathbb{B}$

2: .....wf..... NILNIL

$\vdash \mathbb{B} \in \text{Type}$

3: .....wf..... NILNIL

3. $a \leq\text{z } b = \text{tt}$

$\vdash (a \leq\text{z } b = \text{tt}) \in \mathbb{P}_1$

4: .....wf..... NILNIL

3. $a \leq\text{z } b = \text{tt}$

$\vdash (\uparrow a \leq\text{z } b) \in \mathbb{P}_1$

5: .....wf..... NILNIL

3. $a \leq$z $b =$ tt
$\vdash (a \leq b) \in \mathbb{P}_1$
6: .....wf..... NILNIL

3. $a \leq$z $b =$ tt
$\vdash a \leq$z $b \in \mathbb{B}$
7: .....wf..... NILNIL

3. $a \leq$z $b =$ tt
$\vdash a \in \mathbb{Z}$
8: .....wf..... NILNIL

3. $a \leq$z $b =$ tt
$\vdash b \in \mathbb{Z}$
9: .....wf..... NILNIL

3. $a \leq b$
$\vdash$ (-a) $\leq$z -$b \in \mathbb{B}$
10: .....wf..... NILNIL

3. $a \leq b$
$\vdash \mathbb{B} \in$ Type
11: .....wf..... NILNIL

3. $a \leq b$
4. (-a) $\leq$z -$b =$ tt
$\vdash$ ((-a) $\leq$z -$b =$ tt) $\in \mathbb{P}_1$
12: .....wf..... NILNIL

3. $a \leq b$
4. (-a) $\leq$z -$b =$ tt
$\vdash$ ($\uparrow$(-a) $\leq$z -b) $\in \mathbb{P}_1$
13: .....wf..... NILNIL

3. $a \leq b$
4. (-a) $\leq$z -$b =$ tt
$\vdash$ ((-a) $\leq$ (-b)) $\in \mathbb{P}_1$
14: .....wf..... NILNIL

3. $a \leq b$
4. (-a) $\leq$z -$b =$ tt
$\vdash$ (-a) $\leq$z -$b \in \mathbb{B}$
15: .....wf..... NILNIL

3. $a \leq b$
4. (-a) $\leq$z -$b =$ tt

$\vdash$ (-a) $\in \mathbb{Z}$
16: .....wf..... NILNIL

3. $a \leq b$
4. (-a) $\leq$z -b = tt

$\vdash$ (-b) $\in \mathbb{Z}$
17: .....truecase..... NILNIL

3. $a \leq b$
4. (-a) $\leq$ (-b)

$\vdash$ (-b) = (-a)
18: .....wf..... NILNIL

3. $a \leq b$
4. (-a) $\leq$z -b = ff

$\vdash$ ((-a) $\leq$z -b = ff) $\in \mathbb{P}_1$
19: .....wf..... NILNIL

3. $a \leq b$
4. (-a) $\leq$z -b = ff

$\vdash$ ($\uparrow$(-b) <z (-a)) $\in \mathbb{P}_1$
20: .....wf..... NILNIL

3. $a \leq b$
4. (-a) $\leq$z -b = ff

$\vdash$ ((-b) $<$ (-a)) $\in \mathbb{P}_1$
21: .....wf..... NILNIL

3. $a \leq b$
4. (-a) $\leq$z -b = ff

$\vdash$ ($\uparrow$($\neg_b$(-a) $\leq$z -b)) $\in \mathbb{P}_1$
22: .....wf..... NILNIL

3. $a \leq b$
4. (-a) $\leq$z -b = ff

$\vdash$ (-a) $\leq$z -b $\in \mathbb{B}$
23: .....wf..... NILNIL

3. $a \leq b$
4. (-a) $\leq$z -b = ff

$\vdash$ (-a) $\in \mathbb{Z}$
24: .....wf..... NILNIL

3. $a \leq b$
4. (-a) $\leq$z -b = ff

$\vdash$ (-b) $\in \mathbb{Z}$

25: . . . . .antecedent. . . . . NILNIL

3. $a \leq b$
4. (-a) $\leq$z -b = ff
$\vdash$ True

26: . . . . .wf. . . . . NILNIL

3. $a \leq b$
4. (-a) $\leq$z -b = ff
5. $(\uparrow(\neg_b$(-a) $\leq$z -b)) = ($\uparrow$(-b) <z (-a))
$\vdash \mathbb{P}_1 = \mathbb{P}_1$

27: . . . . .falsecase. . . . . NILNIL

3. $a \leq b$
4. (-b) < (-a)
$\vdash$ (-b) = (-b)

28: . . . . .wf. . . . . NILNIL

3. a $\leq$z b = ff
$\vdash$ (a $\leq$z b = ff) $\in \mathbb{P}_1$

29: . . . . .wf. . . . . NILNIL

3. a $\leq$z b = ff
$\vdash$ ($\uparrow b$ <z a) $\in \mathbb{P}_1$

30: . . . . .wf. . . . . NILNIL

3. a $\leq$z b = ff
$\vdash$ $(b < a) \in \mathbb{P}_1$

31: . . . . .wf. . . . . NILNIL

3. a $\leq$z b = ff
$\vdash$ ($\uparrow(\neg_b a$ $\leq$z b)) $\in \mathbb{P}_1$

32: . . . . .wf. . . . . NILNIL

3. a $\leq$z b = ff
$\vdash$ a $\leq$z b $\in \mathbb{B}$

33: . . . . .wf. . . . . NILNIL

3. a $\leq$z b = ff
$\vdash a \in \mathbb{Z}$

34: . . . . .wf. . . . . NILNIL

3. a $\leq$z b = ff
$\vdash b \in \mathbb{Z}$

35: . . . . .antecedent. . . . . NILNIL

3. $a \leq$z $b =$ ff
$\vdash$ True
36: .....wf..... NILNIL

3. $a \leq$z $b =$ ff
4. $(\uparrow(\neg_b a \leq$z $b)) = (\uparrow b <$z $a)$
$\vdash \mathbb{P}_1 = \mathbb{P}_1$
37: .....wf..... NILNIL

3. $b < a$
$\vdash$ (-a) $\leq$z -$b \in \mathbb{B}$
38: .....wf..... NILNIL

3. $b < a$
$\vdash \mathbb{B} \in$ Type
39: .....wf..... NILNIL

3. $b < a$
4. (-a) $\leq$z -$b =$ tt
$\vdash$ ((-a) $\leq$z -$b =$ tt) $\in \mathbb{P}_1$
40: .....wf..... NILNIL

3. $b < a$
4. (-a) $\leq$z -$b =$ tt
$\vdash$ ($\uparrow$(-a) $\leq$z -b) $\in \mathbb{P}_1$
41: .....wf..... NILNIL

3. $b < a$
4. (-a) $\leq$z -$b =$ tt
$\vdash$ ((-a) $\leq$ (-b)) $\in \mathbb{P}_1$
42: .....wf..... NILNIL

3. $b < a$
4. (-a) $\leq$z -$b =$ tt
$\vdash$ (-a) $\leq$z -$b \in \mathbb{B}$
43: .....wf..... NILNIL

3. $b < a$
4. (-a) $\leq$z -$b =$ tt
$\vdash$ (-a) $\in \mathbb{Z}$
44: .....wf..... NILNIL

3. $b < a$
4. (-a) $\leq$z -$b =$ tt
$\vdash$ (-b) $\in \mathbb{Z}$
45: .....truecase..... NILNIL

3. $b < a$
4. $(\text{-}a) \leq (\text{-}b)$
$\vdash (\text{-}a) = (\text{-}a)$

46: .....wf..... NILNIL

3. $b < a$
4. $(\text{-}a) \leq$z $\text{-}b = \text{ff}$
$\vdash ((\text{-}a) \leq$z $\text{-}b = \text{ff}) \in \mathbb{P}_1$

47: .....wf..... NILNIL

3. $b < a$
4. $(\text{-}a) \leq$z $\text{-}b = \text{ff}$
$\vdash (\uparrow(\text{-}b)$ <z $(\text{-}a)) \in \mathbb{P}_1$

48: .....wf..... NILNIL

3. $b < a$
4. $(\text{-}a) \leq$z $\text{-}b = \text{ff}$
$\vdash ((\text{-}b) < (\text{-}a)) \in \mathbb{P}_1$

49: .....wf..... NILNIL

3. $b < a$
4. $(\text{-}a) \leq$z $\text{-}b = \text{ff}$
$\vdash (\uparrow(\neg_b(\text{-}a) \leq$z $\text{-}b)) \in \mathbb{P}_1$

50: .....wf..... NILNIL

3. $b < a$
4. $(\text{-}a) \leq$z $\text{-}b = \text{ff}$
$\vdash (\text{-}a) \leq$z $\text{-}b \in \mathbb{B}$

51: .....wf..... NILNIL

3. $b < a$
4. $(\text{-}a) \leq$z $\text{-}b = \text{ff}$
$\vdash (\text{-}a) \in \mathbb{Z}$

52: .....wf..... NILNIL

3. $b < a$
4. $(\text{-}a) \leq$z $\text{-}b = \text{ff}$
$\vdash (\text{-}b) \in \mathbb{Z}$

53: .....antecedent..... NILNIL

3. $b < a$
4. $(\text{-}a) \leq$z $\text{-}b = \text{ff}$
$\vdash$ True

54: .....wf..... NILNIL

3. $b < a$
4. $(\text{-}a) \leq\text{z} \text{-}b = \text{ff}$
5. $(\uparrow(\neg_b(\text{-}a) \leq\text{z} \text{-}b)) = (\uparrow(\text{-}b) <\text{z} (\text{-}a))$
$\vdash \mathbb{P}_1 = \mathbb{P}_1$

55: .....falsecase..... NILNIL

3. $b < a$
4. $(\text{-}b) < (\text{-}a)$
$\vdash (\text{-}a) = (\text{-}b)$

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http://www.nuprl.org/FDLcontent/p0_942988_/p121_72896_{minus_imax}1_0.html